Nonlinear reflection of internal gravity wave onto a slope

Combining lab experiments, 2D and 3D simulations

K.Raja[1], J.Sommeria[1], C.Staquet[1], M.Leclair[2], N.Grisouard[3] & L.Gostiaux[4]

[1]*LEGI, UGA*, [2]*ETH Zurich*, [3]*UToronto*, [4]*LMFA, Lyon*

Contact Information:
Keshav Raja
LEGI
Universite Grenoble Alpes
Grenoble, France
Email: **keshav.raja@legi.grenoble-inp.fr**

Introduction and objective

The nonlinear reflection of internal waves from a sloping boundary is studied using laboratory experiments (carried out on the Coriolis Platform at Grenoble) and, 2D and 3D numerical simulations (performed using a non-hydrostatic code). The interaction of the incident and reflected waves produce, an irreversible wave induced mean flow which grows in time and is localised in the interacting region. **The growth and energetics of this wave-induced mean flow is studied.**

Experimental setup

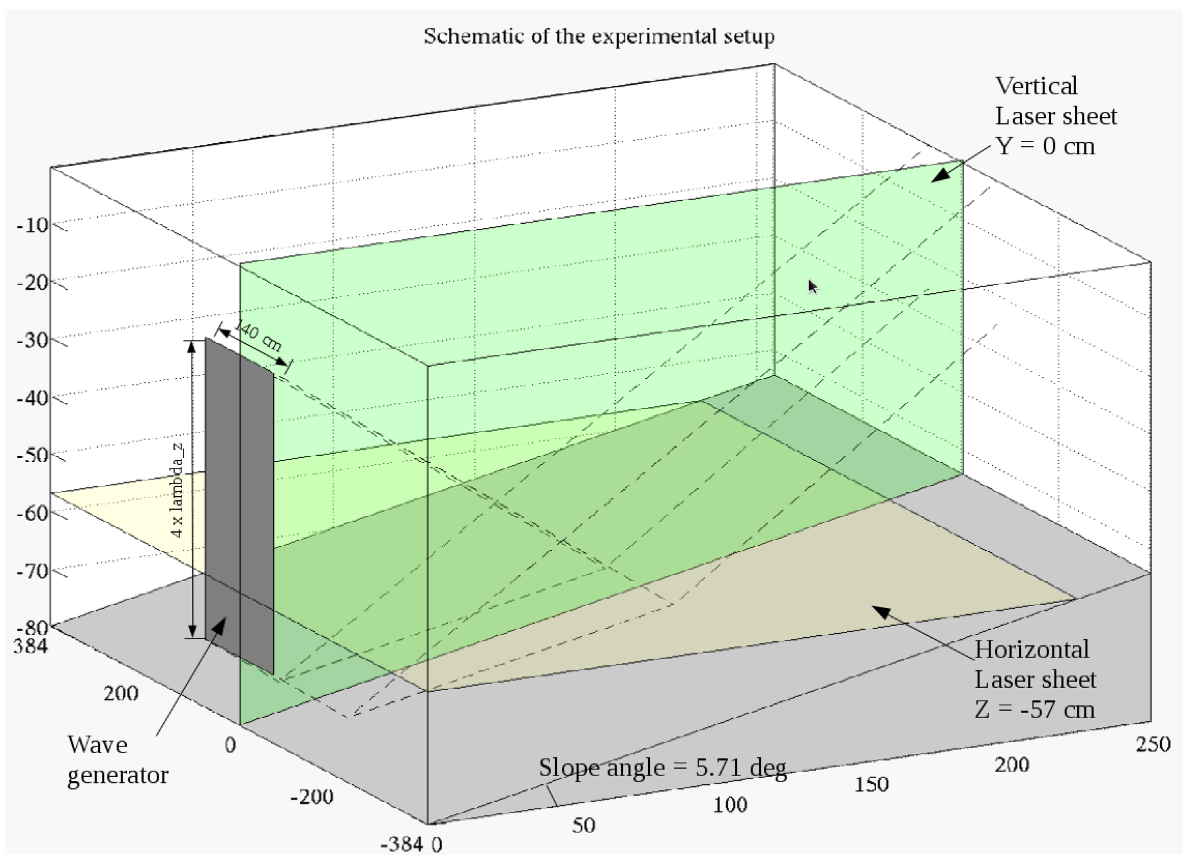

Figure 1: Schematic of the **experiment at Coriolis platform, LEGI**. In the experiment, a plane wave is produced using a wave generator and is made to reflect normally on a sloping bottom in a uniformly stratified fluid. Velocity fields are obtained by PIV using a horizontal and a vertical laser sheet.
Numerical simulations done using a non-hydrostatic model mimick the lab experiment, with a resolution of 1cm in horizontal and 0.5cm in vertical direction. The boundary condition on the slope is free-slip.

Horizontal velocity field

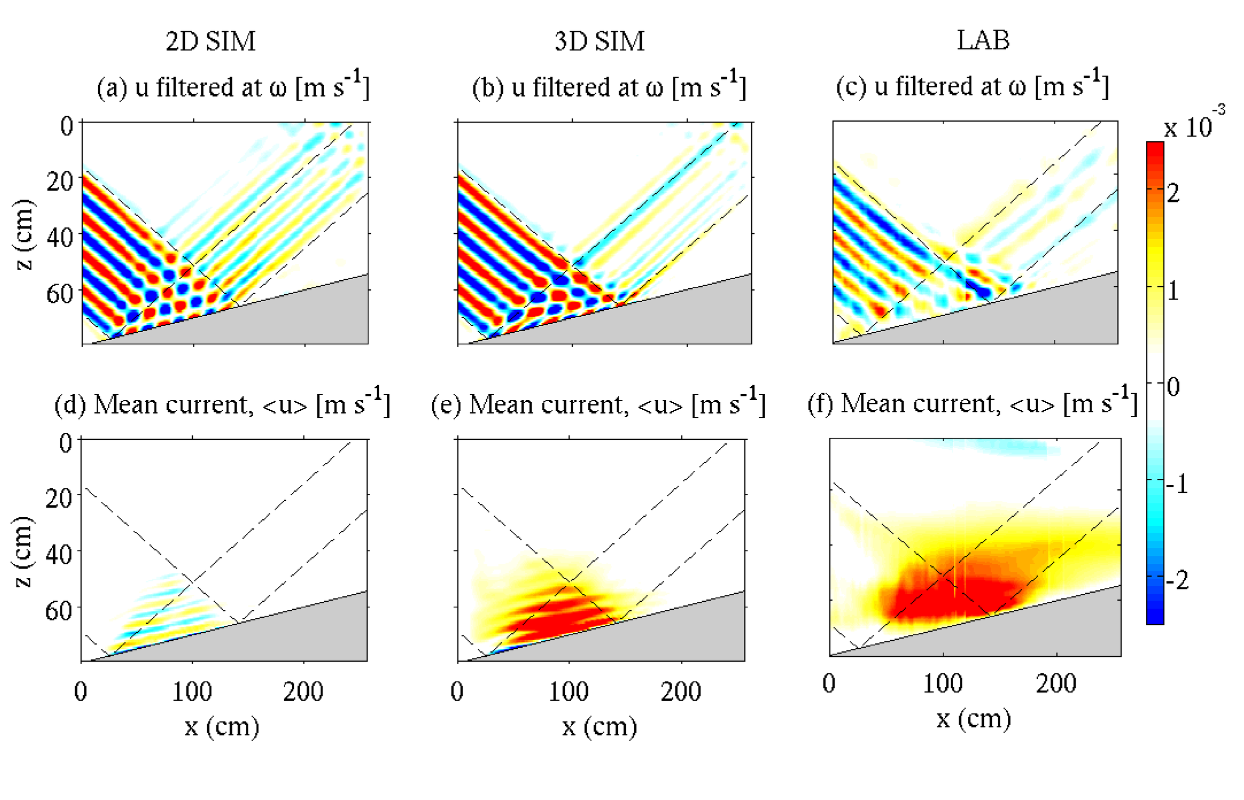

Figure 2: Zonal velocity fields in the vertical section (y = 0 cm) filtered over 17-20 wave periods at the forcing frequency (a, b, c); and Eulerian mean currents (d, e, f).

Wave induced mean flow

The wave induced mean flow in lab experiments and 3D simulation compare well, while in 2D simulation, the theoretically predicted spatially periodic mean flow is found. The evolution of kinetic energy averaged in the interaction region for different harmonics and the mean flow is shown in Figure 3.

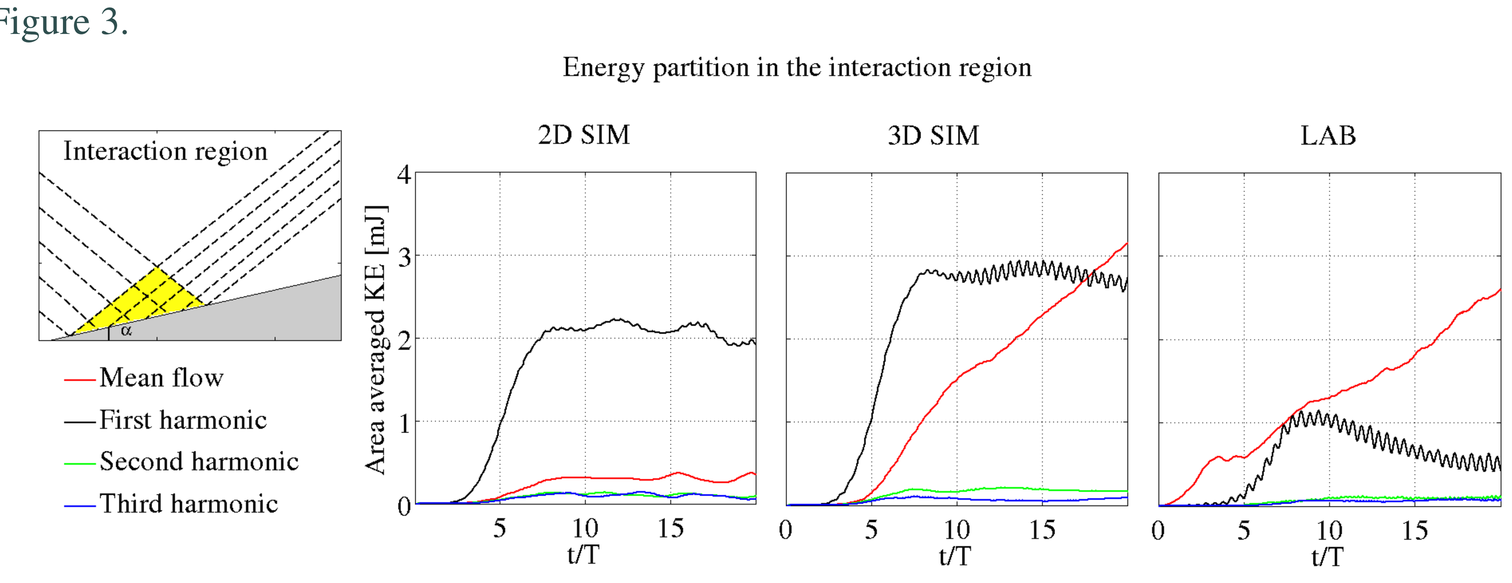

Figure 3: Kinetic energy partition in the mean flow and different harmonics in the interaction region.

The KE associated with the mean flow grows strongly and exceed that of primary wave in the lab experiment and 3D simulation, while it is weak in 2D simulation. The mean flow in lab and 3D simulation depend primarily on nonlinear and dissipative effects, and therefore it is cumulative in time and irreversible. Hence, there should be an associated Lagrangian mean flow in this case.

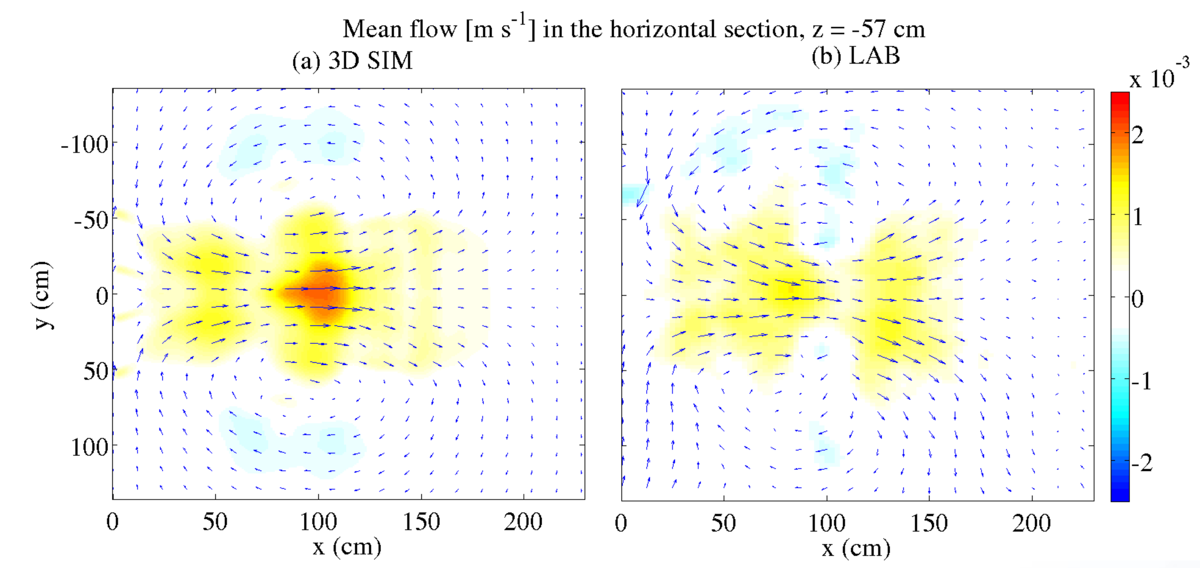

Figure 4: Mean flow in the horizontal section, z = -57 cm.

The wave induced mean flow in the lab experiment and 3D simulation also recirculate in the horizontal plain, generating a strong vertical vorticity field.

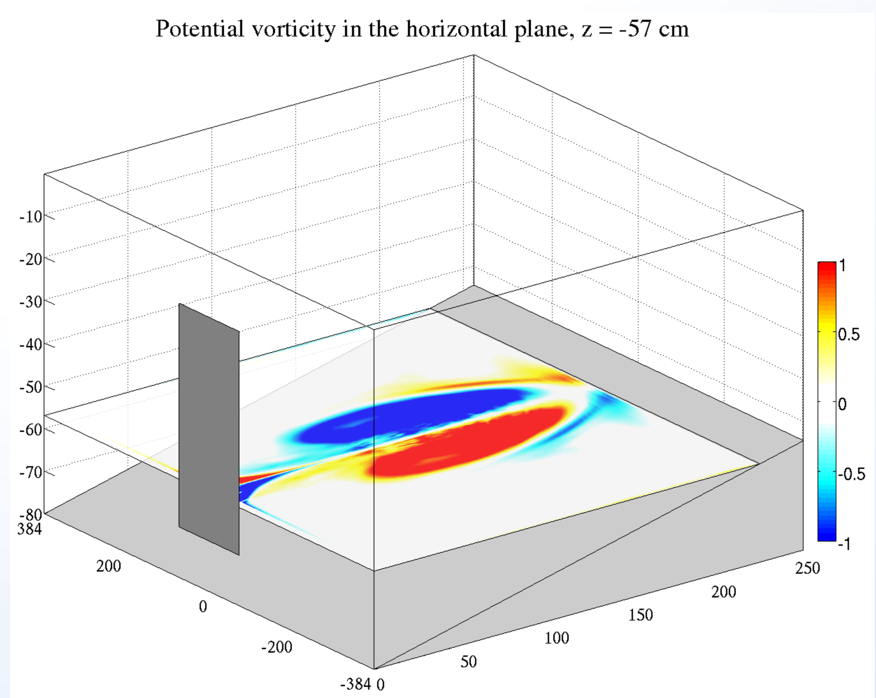

Figure 5: Potential vorticity in the horizontal section, z = -57 cm from the 3D simulation.

Mean current acceleration theory

The acceleration of the mean flow due to the interaction of incident and reflected wave can be calculated analytically, from substituting viscous linear wave solutions to the Reynolds stress term in the momentum equation.

M. Leclair derived the expression for mean current acceleration during the reflection. According to his theory, the acceleration term constitutes of A_E, acceleration of a purely Eulerian mean flow (without net mass transport) associated with 2D case, and A_L, acceleration of a non reversible mean flow which is proportional to the kinematic viscosity.

$$A = -\left(\frac{\partial \overline{uu}}{\partial x} + \frac{\partial \overline{uw}}{\partial z}\right) = \underbrace{A_E}_{\text{2D periodic mean flow}} + \underbrace{A_L}_{\text{Irreversible mean flow}}$$

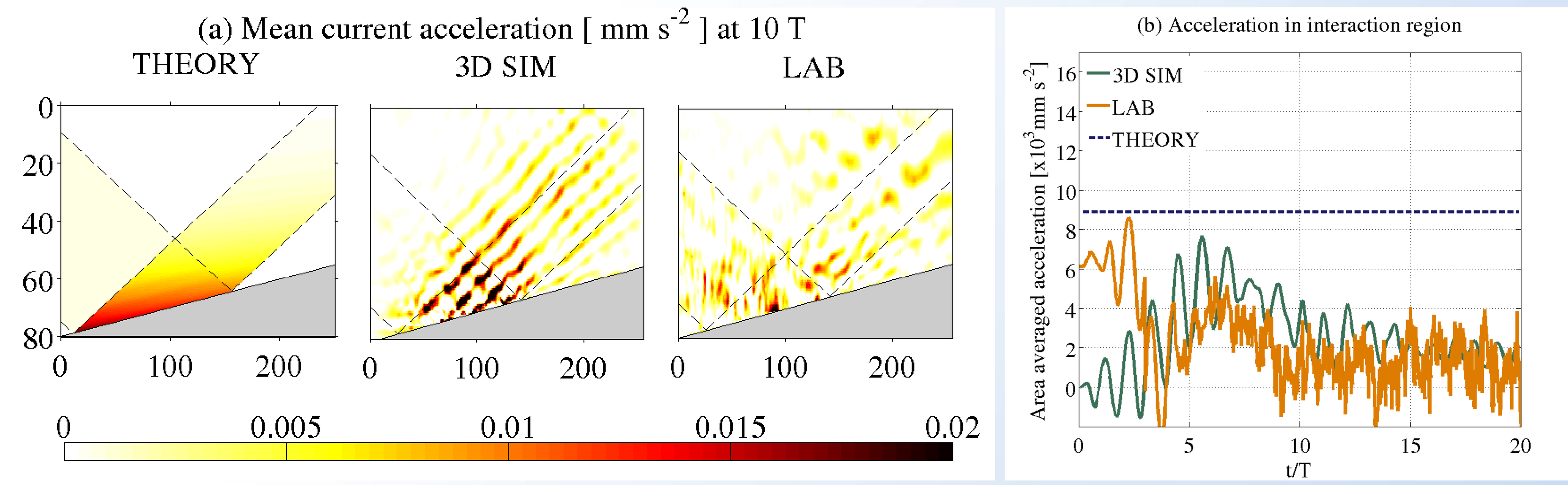

Figure 6: Comparison of the mean current acceleration from theory, simulation and lab experiments, for time 10 T (a); Evolution of the mean current acceleration in the interaction region with time (b).

The acceleration is dominated by the reflected wave and is maximum in the interaction region. The theoretical mean current acceleration and the observed acceleration in lab and simulation, are in the same order of magnitude, even though the theoretical prediction over-estimates the value, pointing towards a scope for improvement.

Conclusions

- Internal waves reflection produces a strong irreversible mean flow, caused by the combined effect of nonlinear and viscous terms. This is different from the spatially periodic 2D mean flow predicted by theory before.
- The strong irreversible 3D mean flow refracts the wave field in the interaction region, enhancing the focusing of the reflected wave, finally leading to its breaking.
- The 3D mean flow recirculates in the horizontal plane producing a dipole vortex structure.
- The acceleration of the mean current is due to the combined effect of nonlinearity and viscosity, and is dominated by the reflected wave and is proportional to the viscosity.

Forthcoming Research

- Energy budget in 3D simulations to quantify the transfer of energy from the primary wave to higher harmonics, mean flow and small scale processes.
- Improve the theoretical model for mean acceleration. Test the theory for different amplitudes of incident wave.
- Higher resolution simulations; study boundary effects.
- Effects of rotation, boundary layer.

Publications

N. Grisouard, M. Leclair, L. Goatiaux and C. Staquet 2013. Large scale energy transfer from an internal gravity wave reflecting on a simple slope *IUTAM Symposium Procedia* 8 119-128

A Javam, J, Imberger and S. W. Armfield 1999. Numerical study of internal wave reflection from sloping boundaries *Journal of Fluid Mechanics. vol. 396, pp. 183-201*

M. Leclair, K. Raja and C. Staquet 2016. Nonlinear reflection of a two-dimensional finite-width internal gravity wave onto a slope *Journal of Fluid Mechanics. in preparation*