Nonlinear reflection of internal gravity wave beam on a slope

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Introduction and objective

The nonlinear reflection of internal waves from a sloping boundary is studied using laboratory experiments (carried out on the Coriolis Platform at Grenoble) as well as 2D and 3D numerical simulations (performed using a non-hydrostatic model). The interaction of the incident and reflected waves produces an irreversible wave-induced mean flow which grows in time and is localised in the interacting region. **The generation and evolution of this wave-induced mean flow is presented in this poster, within an Eulerian framework.**

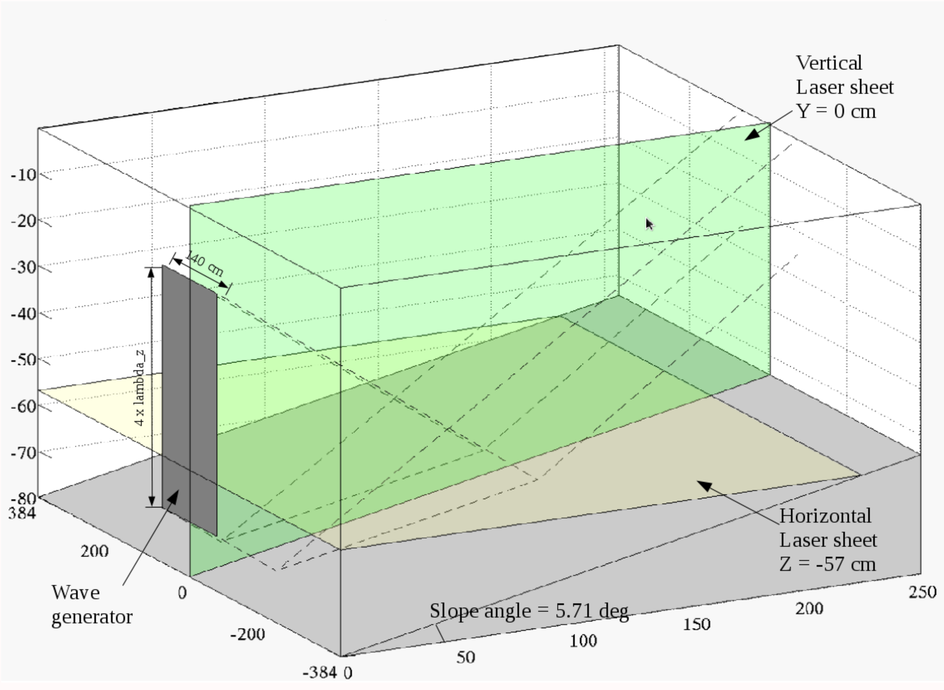

Figure 1: Domain of interest in the **experiment at Coriolis platform, LEGI**. In the experiment, a plane wave is produced using a wave generator ($\lambda_z = 0.12m$) and is made to reflect normally on a sloping bottom in a uniformly stratified fluid. Velocity fields are obtained by PIV using a horizontal and a vertical laser sheet.
Numerical simulations done using a non-hydrostatic model mimick the lab experiment using the domain shown, with a resolution of 1cm in horizontal and 0.5cm in vertical direction. The boundary condition on the slope is free-slip.

Horizontal velocity field and wave-induced mean flow

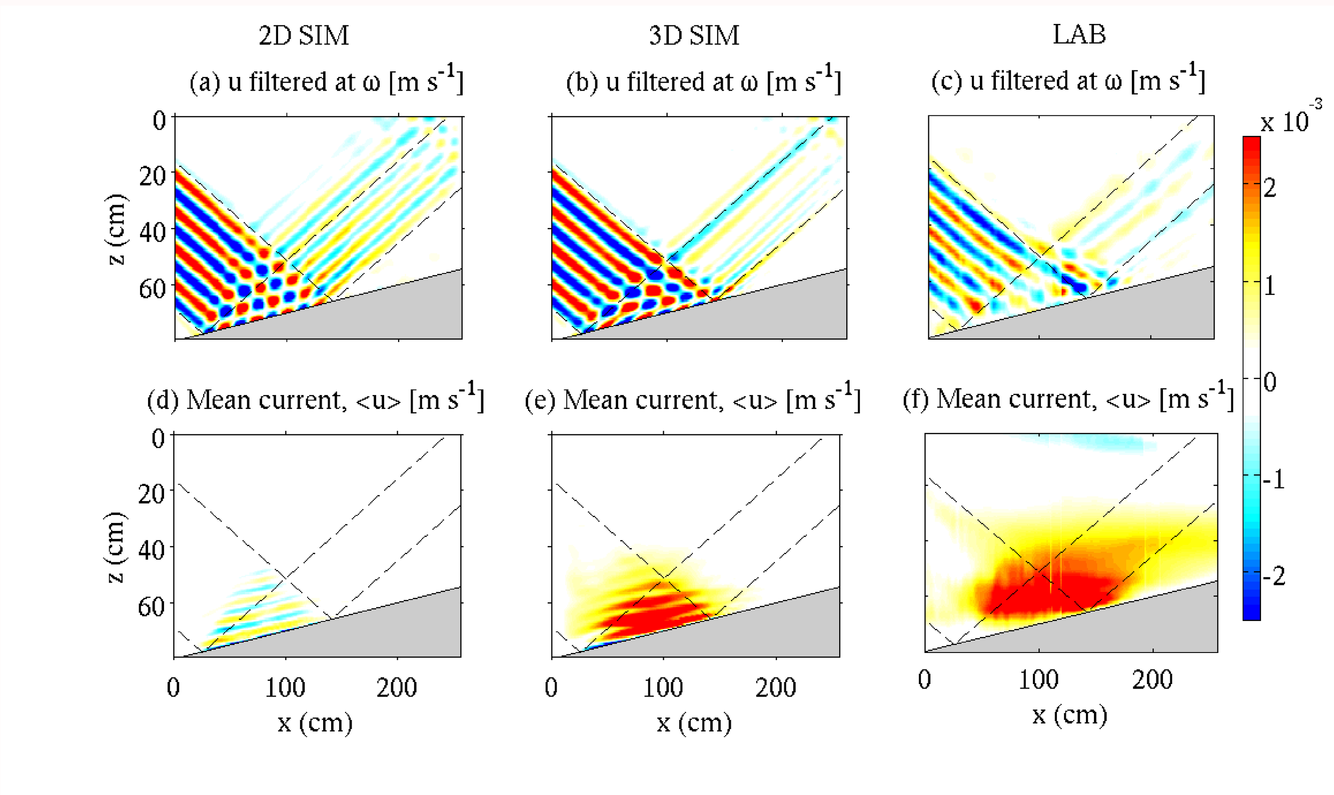

Figure 2: Zonal velocity fields in the vertical section (y = 0 cm) filtered over 19-21 wave periods at the forcing frequency (a, b, c); and Eulerian mean currents (d, e, f).

The wave-induced mean flow in laboratory experiments and 3D simulation compare well, while in 2D simulation, the theoretically predicted (Thorpe 1987) spatially periodic Eulerian mean flow is found. The Eulerian mean flow opposes the Stokes drift in 2D. Hence the total Lagrangian mean flow, which is the sum of the Eulerian mean flow and the Stokes drift (Longuet-Higgins 1969), is zero. The mean flow in experiment and 3D simulation depends primarily on nonlinear and dissipative effects, and therefore it is cumulative in time and irreversible. Hence, there should be an associated Lagrangian mean flow in this case. The amplitude of the mean flow grows and exceeds that of primary wave in the experiment and 3D simulation while it is weak in 2D simulation.

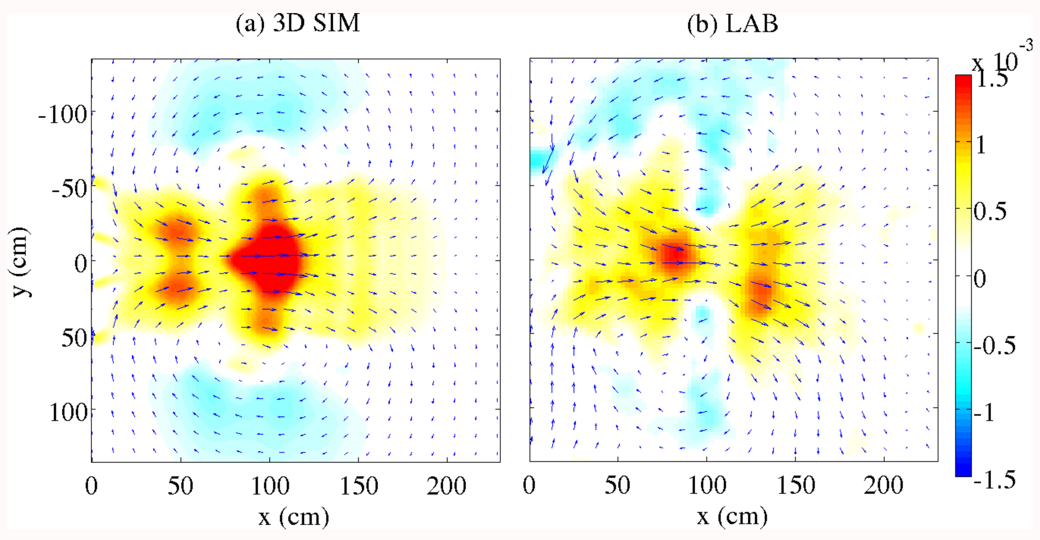

Figure 3: Mean flow in the horizontal section, z = -57 cm, averaged over 19-21 wave periods.

The wave-induced mean flow in the experiment and 3D simulation also recirculates in the horizontal plane, generating a strong vertical vorticity field.

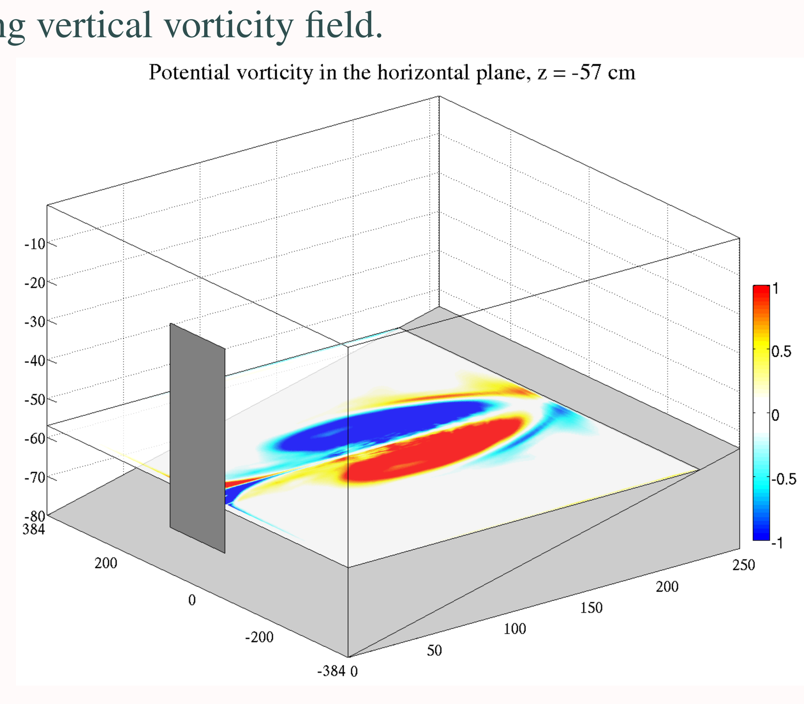

Figure 4: Potential vorticity in the horizontal section, z = -57 cm from the 3D simulation after 40 wave periods.

The finite width of the wave beam in transverse (y-) direction creates variation of the wave field in that direction. The vertical vorticity is forced by the derivatives of the Reynolds stress divergence with respect to y. Thus, the re-circulation of the mean flow (and the dipole vorticity structure in the horizontal plane) is essentially due to the finite width of the wave beam in transverse direction.

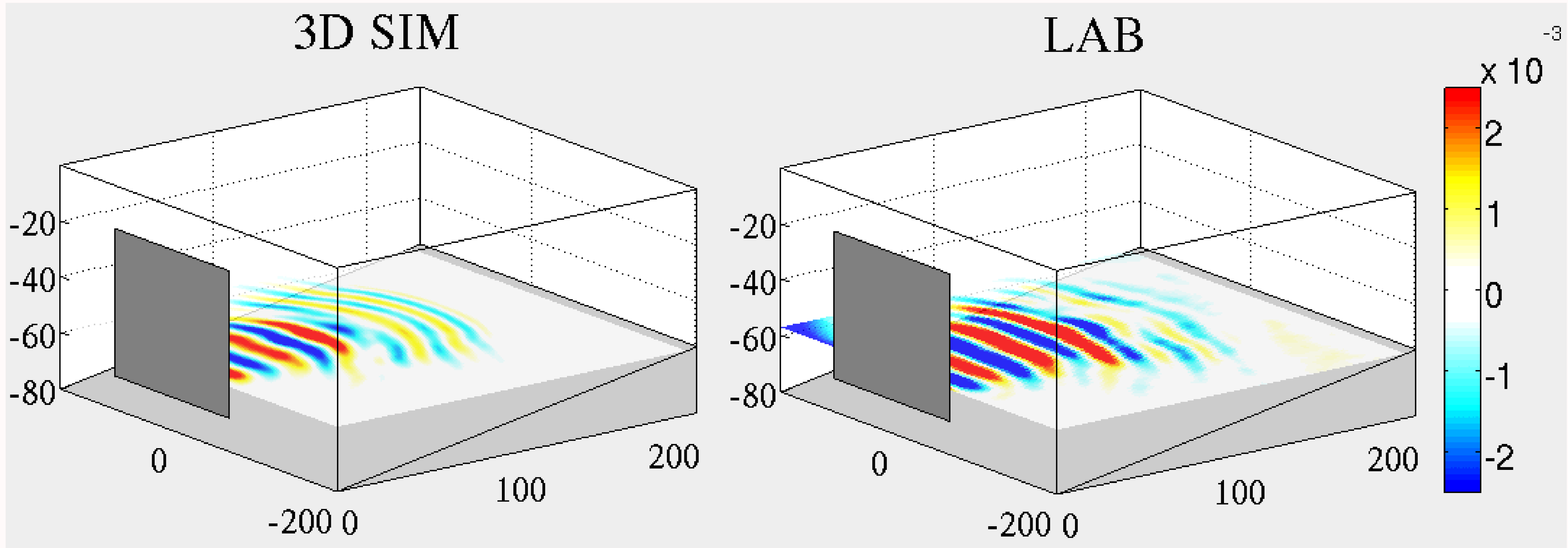

Figure 5: Zonal velocity fields in the horizontal section (z = -57 cm) filtered over 19-21 wave periods at the forcing frequency. Note that the velocity field is varying in the y-direction.

Energy budget

The energy balance equation can be written for the filtered wave and mean fields.

$$\frac{\partial}{\partial t}(TME_{wave}) = Influx_{wave} - Efflux_{wave} + Dissipation_{wave} + (Sink) \qquad (1)$$

$$\frac{\partial}{\partial t}(TME_{mean}) = Influx_{mean} - Efflux_{mean} + Dissipation_{mean} + (Source) \qquad (2)$$

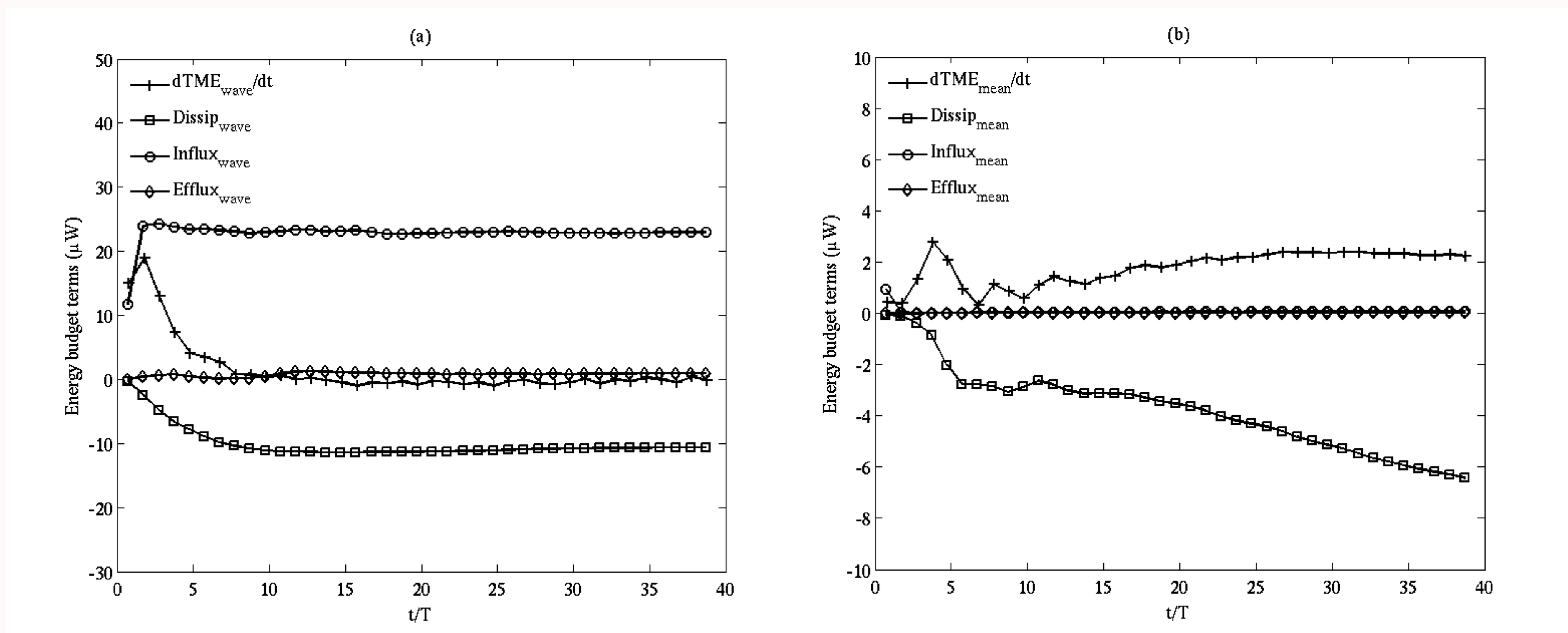

Figure 6: Energy budget for (a) the filtered wave field and (b) the mean field, in the whole 3D domain except the boundaries. The different terms are not balanced indicating a loss of energy from the wave field and a gain of energy to the mean field.

Assuming the source of energy for the mean flow is coming solely from the fundamental wave field, we can estimate the flux of energy from the wave to the mean flow and to higher harmonics, using the terms in the equations above.

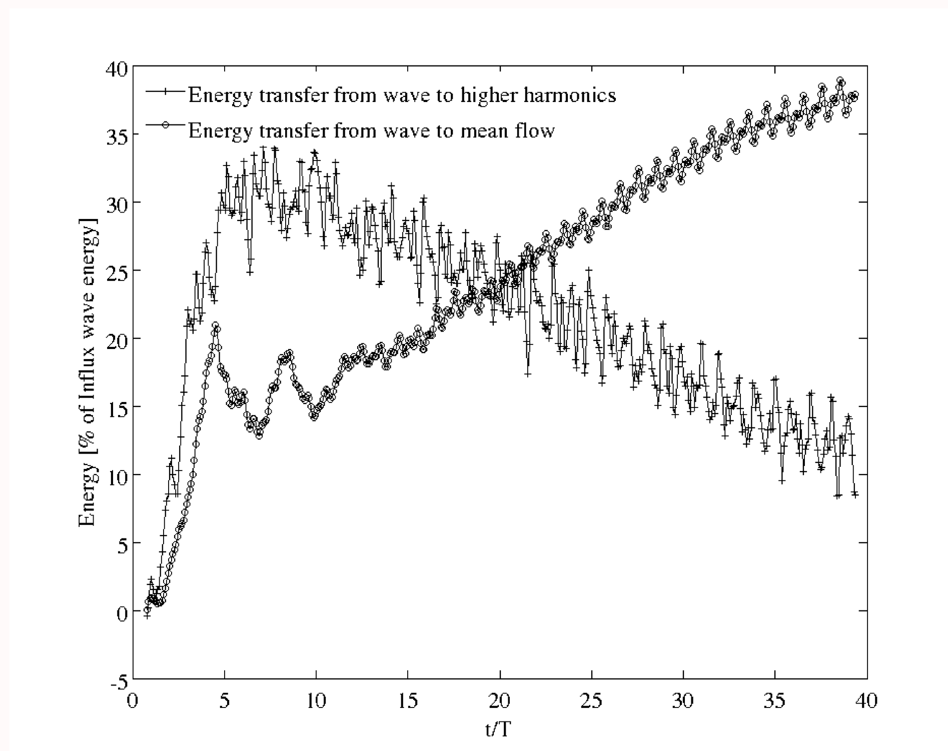

Figure 7: Energy transfer from the wave to the higher harmonics and the mean flow, normalised by the influx of wave energy into the domain.

Conclusions

- Internal wave beam reflection produces a strong irreversible 3D Eulerian mean flow, caused by the combined effect of nonlinear and viscous terms.
- The 3D mean flow is a combination of a spatially periodic Eulerian mean flow close to the slope due to wave interactions and a 3D mean flow recirculating in the horizontal plane due to the finite width of the wave beam in transverse direction.
- The energy flux from the wave to the mean flow increases with time and reaches close to 38% (of influx wave energy) by the end of 40 wave periods.

References

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